

## Test 3: Projectiles

Equations and Constants:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

$$\bar{v} = \frac{1}{2}(v_i + v_f)$$

2013

$$|g| = 10 \text{ m/s}^2$$

85 total

$$x = \frac{1}{2}at^2 + v_i t + x_i$$

$$v = at + v_i$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$a_c = \frac{v^2}{r}$$

$$2 \sin \theta \cos \theta = \sin 2\theta$$

**Multiple Choice: Choose the letter of the best answer. 3 points each.**

Problems 1 to 4 refer to the following:

$$v_{ji} =$$

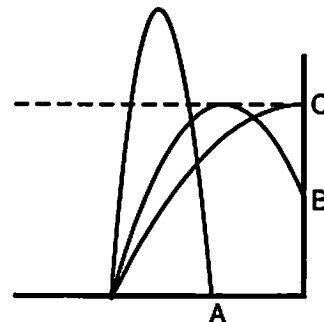
A projectile is launched with a velocity of 40 m/s at an angle of  $30^\circ$  above the horizontal from the ground on a flat, level field on some other planet. It lands 85 meters away from its launch site.

$$85 = \frac{40^2 \sin(60^\circ)}{g}$$

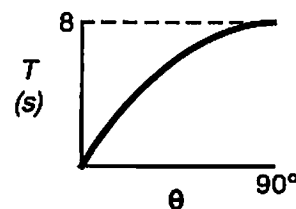
- D What is the acceleration due to gravity on this planet?  
a.  $2.45 \text{ m/s}^2$ .    b.  $4.25 \text{ m/s}^2$ .    c.  $9.41 \text{ m/s}^2$ .    d.  $16.3 \text{ m/s}^2$ .    e.  $18.8 \text{ m/s}^2$ .
- B How fast is the projectile going at its maximum height?  
a. 0 m/s.    b. 34.6 m/s.    c. 20 m/s.    d. 40 m/s.    e. 28.3 m/s.
- C How fast would a second projectile have to be launched, with the same initial angle, to land twice as far away?  
a. 40 m/s.    b. 80 m/s.    c. 56.6 m/s.    d. 69.3 m/s.    e. 6.3 m/s.
- A Which of the following initial velocities would reach the same maximum height?  
a.  $10\mathbf{i} + 20\mathbf{j} \text{ m/s}$ .    b.  $20\mathbf{i} + 10\mathbf{j} \text{ m/s}$ .    c.  $50 \text{ m/s} @ 45^\circ$     d.  $40 \text{ m/s} @ 60^\circ$     e.  $20\mathbf{i} + 34.6\mathbf{j} \text{ m/s}$ .

Problems 5 and 6 refer to the following:

Three projectiles are fired across level ground from the same spot. One hits the ground and two others hit the side of a building as shown in the diagram. The two that hit the building had the same maximum height.

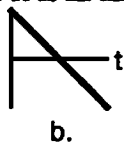
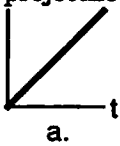


- B Rank the projectiles, from largest to smallest, by initial vertical velocity.  
a.  $A > B > C$ .    b.  $A > B = C$ .    c.  $A > C > B$ .  
d. all the same.    e. can't tell from the diagram.
- A Rank the projectiles, from largest to smallest, by time of flight.  
a.  $A > B > C$ .    b.  $A > B = C$ .    c.  $A > C > B$ .    d. all the same.  
e. can't tell from the diagram.
- C Two identical balls (A & B) are at the same initial height above a level field. Ball A is launched horizontally and at the same time, ball B is simply dropped. Which one hits the ground first?  
a. A.    b. B.    c. neither, they hit at the same time.
- A A projectile is always launched with the same speed, but at different initial angles across a level field. The total time in the air was recorded. The results are shown in the graph. What was the initial speed of the projectile?  
a. 40 m/s.    b. 28.3 m/s.    c. 56.6 m/s.  
d. 80 m/s.    e. 90 m/s.



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9. B Which of the following graphs best represents the vertical component of the velocity of a projectile while it is in the air?



Problems 10 and 11 refer to the following:

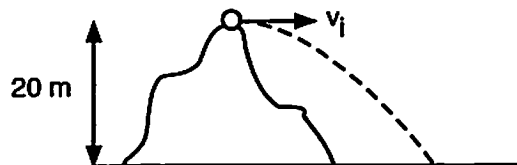
A projectile is launched from the ground with the same speed across a level floor at initial angles of  $A = 30^\circ$ ,  $B = 45^\circ$ ,  $C = 55^\circ$  and  $D = 60^\circ$ .

10. C Rank the projectiles according to how far away they landed (greatest to least.)  
 a.  $D > C > B > A$ .    b.  $B > D = A > C$ .    c.  $B > C > D = A$ .    d.  $D > B = C > A$ .
11. A Rank the projectiles according to their maximum heights (greatest to least.)  
 a.  $D > C > B > A$ .    b.  $A > B > C > D$ .    c.  $B > C > D = A$ .    d.  $D > B = C > A$ .

Problems 12 and 13 refer to the following:

A projectile has an initial velocity of  $25\mathbf{i} + 15\mathbf{j}$  m/s.

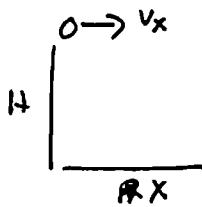
12. B In what direction is it traveling?  
 a.  $59^\circ$ .    b.  $31^\circ$ .    c.  $45^\circ$ .    d.  $37^\circ$ .    e.  $53^\circ$ .
13. A How many seconds will it take to reach its maximum height?  
 a. 1.5 s.    b. 2.5 s.    c. 2.9 s.    d. 3.0 s.    e. 5.8 s.
14. E A tennis ball is thrown up in the air at some initial angle from some initial height. At which of the following points is the acceleration of the tennis ball the greatest?  
 a. At the instant after the tennis ball is launched.  
 b. At the peak of the trajectory.  
 c. At the instant before the tennis ball hits the ground.  
 d. The acceleration is maximized at both (a) and (c).  
 e. The acceleration is constant.



15. D A ball is thrown horizontally from the top of a 20 meter high hill. It strikes the ground at an angle of  $45^\circ$ . With what speed was it thrown?  
 a. 40 m/s.    b. 32 m/s.    c. 28 m/s.    d. 20 m/s.    e. 14 m/s.

**Test 3: Projectiles****Problem Solving: Show all work. 10 points each.**

16. A student fires a metallic sphere horizontally off a table with a speed of 2.5 m/s. It lands 1.1 meter away (horizontally) from the edge of the table. How high was the table?



$$X = V_x t$$

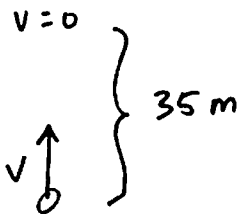
$$1.1 = (2.5)t$$

$$t = \underline{\underline{0.44\text{ s}}}$$

$$H = \frac{1}{2} (10) (.44)^2 \quad (H = \frac{1}{2} g t^2)$$

$$H = \underline{\underline{0.97\text{ m}}}$$

17. A tennis ball can be fired at any initial angle, but always the same speed on a flat level field. The absolute highest the tennis ball can reach is 35 meters. What is the absolute longest distance the tennis ball could be launched?



Straight up = Max possible height.

$$V_f^2 = V_i^2 + 2a \Delta x$$

$$0 = V_i^2 + 2(-10)(35)$$

$$V_i^2 = 700$$

$$V_i = \underline{\underline{26.5\text{ m/s}}}$$

$$\text{or } 35 = \frac{1}{2} (10) t^2$$

$$t^2 = 7$$

$$t = 2.65$$

$$v_{fx} = 10(2.65)$$

$$V_i = 26.5\text{ m/s}$$

Max Range  $\theta = 45^\circ$

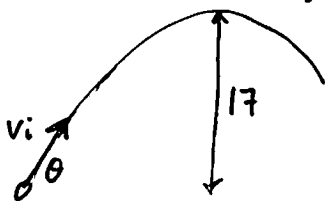
$$R = \frac{(26.5)^2 \sin(2 \cdot 45)}{10}$$

$$R = \underline{\underline{70\text{ m}}}$$

Said  $V_i = V_{yi}$   
 $\theta = 45^\circ$   $V_x = V_{xi}$   
 (i.e. assumed initial  
 shot was  $\theta = 45^\circ$ )  
 (-4)

### Test 3: Projectiles

18. A projectile is fired at an angle of  $65^\circ$  above the horizontal. It reaches a maximum height of 17 meters. How many seconds would it take to travel 20 meters horizontally?



$$y = \frac{1}{2} g t^2$$

$$17 = \frac{1}{2} (10) t^2$$

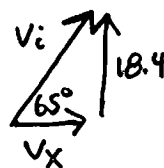
$$t^2 = 3.4$$

$$t = 1.84 \text{ s}$$

$$v_y = g t$$

$$= (10)(1.84)$$

$$v_{y_i} = 18.4 \text{ m/s}$$



$$\tan 65 = \frac{18.4}{v_x}$$

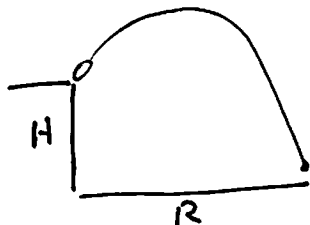
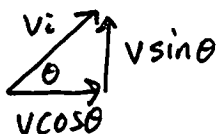
$$v_x = 8.6 \text{ m/s}$$

$$x = v_x t$$

$$20 = (8.6) t$$

$$t = 2.33 \text{ sec.}$$

19. A projectile is launched from the top of a cliff of height  $H$  at an initial angle of  $\theta$ . It lands a horizontal distance  $R$  away from the base of the cliff. What was the initial speed of the projectile?



$$R = v \cos \theta t$$

$$0 = -\frac{1}{2} g t^2 + v \sin \theta t + H$$

$$\text{So } t = \frac{R}{v \cos \theta}$$

$$\text{So } 0 = -\frac{1}{2} g \left( \frac{R}{v \cos \theta} \right)^2 + v \sin \theta \left( \frac{R}{v \cos \theta} \right) + H$$

$$0 = \frac{-g R^2}{2 v^2 \cos^2 \theta} + R \tan \theta + H$$

$$2 v^2 \cos^2 \theta = \frac{H A / R \tan \theta}{g R^2} \frac{g R^2}{R \tan \theta + H}$$

$$v^2 = \frac{g R^2}{2 \cos^2 \theta (H + R \tan \theta)}$$

$$\frac{\frac{m}{s^2} \cdot m^2}{m} \checkmark$$

units ok.

Oops. So  $v = \sqrt{\quad}$

$$\text{or } \frac{g R^2}{2 \cos^2 \theta H + R \sin 2\theta}$$

$$2 \cos^2 \theta R \sin \theta = R \sin 2\theta$$

side 4